

TEMPERATURE STRUCTURE AND X-RAY EMISSION OF A THIN ACCRETION DISK AROUND A STELLAR-MASS BLACK HOLE ($M = 10 M_{\odot}$)

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Abstract. Within the Shakura–Sunyaev model, the radial temperature profile, bolometric luminosity and Keplerian frequency of a thin accretion disk around a $10 M_{\odot}$ Schwarzschild black hole are calculated. For an accretion rate of $10^{-8} M_{\odot}/\text{yr}$ the maximum disk temperature is $T_{\text{max}} \approx 3.46$ million K, the luminosity is $L \approx 4.72 \times 10^{37} \text{ erg/s}$ ($L/L_{\text{Edd}} \approx 0.038$), and the Keplerian frequency at the inner edge is ≈ 220 Hz.

Keywords: accretion disk, black hole, Shakura–Sunyaev model, X-ray emission

1. Introduction

Accretion disks around black holes and neutron stars are the main sources of the high-energy radiation observed in X-ray binaries. Matter in the disk loses angular momentum through viscosity and spirals inward, releasing gravitational potential energy as thermal radiation. The aim of this work is to obtain quantitative estimates of the disk temperature, luminosity and dynamical characteristics using the classical analytic model.

2. Theoretical Background

We consider a geometrically thin, optically thick, steady disk in Keplerian rotation around a non-rotating black hole of mass M . Matter accretes at a constant rate $\dot{M}$, and viscous torques, parametrized by the α -prescription [1], transport angular momentum outward. The energy released by viscous dissipation per unit area of one disk face is [3]

$$D(r) = \frac{3GM\dot{M}}{8\pi r^3} \left[1 - \sqrt{\frac{R_{\text{in}}}{r}} \right] \quad (1)$$

where $R_g = GM/c^2$ and $R_{\text{in}} = 6R_g$ is the innermost stable circular orbit of a Schwarzschild black hole. If each face radiates as a blackbody, $\sigma T^4 = D(r)$, and the temperature profile is

$$T(r) = \left[\frac{3GM\dot{M}}{8\pi\sigma r^3} \left(1 - \sqrt{\frac{R_{\text{in}}}{r}} \right) \right]^{1/4} \quad (2)$$

Far from the inner edge $T \propto r^{-3/4}$. The profile peaks at $r = (49/36)R_{\text{in}} \approx 8.17R_g$, where $T_{\text{max}} = 0.488T_*$ and $T_* = [3GM\dot{M}/(8\pi\sigma R_{\text{in}}^3)]^{1/4}$; because $R_{\text{in}} \propto M$, this gives $T_{\text{max}} \propto (\dot{M}/M^2)^{1/4}$. Integrating the dissipation over both faces gives the disk luminosity

$$L = \int_{R_{\text{in}}}^{\infty} 4\pi r D(r) dr = \frac{GM\dot{M}}{2R_{\text{in}}} = \frac{\dot{M}c^2}{12} \quad (3)$$

The efficiency $\eta = 1/12 \approx 0.083$ corresponds to the pseudo-Newtonian approximation; the fully relativistic Schwarzschild value is $\eta \approx 0.057$ [2]. Since the disk is a superposition of blackbodies of different temperatures, its spectrum is the multicolor blackbody [4]

$$L_v = 4\pi^2 \int_{R_{in}}^{\infty} B_v[T(r)] r dr \quad (4)$$

where $B_v(T)$ is the Planck function. The thin-disk assumption is checked with the Eddington luminosity and the Keplerian frequency,

$$L_{Edd} = \frac{4\pi G M m_p c}{\sigma_T} \quad (5)$$

$$f_K(r) = \frac{1}{2\pi} \sqrt{\frac{GM}{r^3}} \quad (6)$$

In the inner region, where radiation pressure and electron scattering (opacity $\kappa_{es} = 0.04 \text{ m}^2/\text{kg}$) dominate, the half-thickness of the disk is [1,3]

$$H(r) = \frac{3\kappa_{es}\dot{M}}{8\pi c} \left[1 - \sqrt{\frac{R_{in}}{r}} \right] \quad (7)$$

This expression is an upper limit, because gas pressure reduces H at larger radii. The thin-disk model requires $H/r \ll 1$ and, equivalently, L well below L_{Edd} .

3. Calculations and Results

Input parameters: $M = 10 M_{\odot}$, $\dot{M} = 10^{-8} M_{\odot}/\text{yr} = 6.30 \times 10^{14} \text{ kg/s}$. We obtain $R_g = 14.77 \text{ km}$, $R_{in} = 88.60 \text{ km}$, $T_* = 7.09 \times 10^6 \text{ K}$ and $T_{max} = 3.46 \times 10^6 \text{ K}$ at $r \approx 120.6 \text{ km}$. This corresponds to $kT_{max} \approx 0.298 \text{ keV}$, with the Planck peak ($h\nu \approx 2.82kT$) at $\approx 0.84 \text{ keV}$. The luminosity is $L = 4.72 \times 10^{37} \text{ erg/s}$ against $L_{Edd} = 1.26 \times 10^{39} \text{ erg/s}$, so $L/L_{Edd} = 0.0375$; the Keplerian frequency at R_{in} is $f_K \approx 220 \text{ Hz}$.

With the hardening factor $f \approx 1.7$ [5] the color temperature is $T_{col} = fT_{max} \approx 5.88 \text{ million K}$ ($kT_{col} \approx 0.51 \text{ keV}$). For the disk thickness, $H_0 = 3\kappa_{es}\dot{M}/(8\pi c) = 10.0 \text{ km}$; the ratio H/r reaches its maximum 0.0168 at $r = 2.25R_{in} = 13.5R_g$ (at r_{max} it is 0.0119), so $H/r \ll 1$ throughout.

r/R_g	$r, \text{ km}$	$T(r), \text{ K}$	$kT, \text{ eV}$
6.50	96	2.97×10^6	256
8.17	121	3.46×10^6	298
10	148	3.33×10^6	287
20	295	2.36×10^6	203
50	738	1.30×10^6	112
100	1477	8.02×10^5	69
1000	14767	1.50×10^5	13

Table 1. Disk temperature versus radius ($M = 10 M_{\odot}$, $\dot{M} = 10^{-8} M_{\odot}/\text{yr}$).

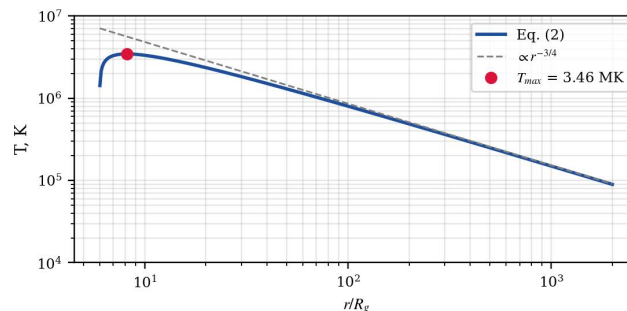


Figure 1. Disk temperature profile: solid line — Eq. (2); dashed — the $r^{-3/4}$ law.

The dependence on the accretion rate is shown in Table 2. Since $T_{\max} \propto \dot{M}^{1/4}$ and $L \propto \dot{M}$, the Eddington limit $L = L_{\text{Edd}}$ is reached at $\dot{M}_{\text{Edd}} = L_{\text{Edd}}/(\eta c^2) = 2.66 \times 10^{-7} M_{\odot}/\text{yr}$. There $H_0 \approx 267$ km, about 3.0 times R_{in} , so the thin-disk description breaks down near this rate and thick (slim) disk models are required.

$\dot{M}$, $M_{\odot}/\text{yr}$	L , erg/s	L/L_{Edd}	$T_{\max}$, K	$kT_{\max}$, keV
10^{-9}	4.72×10^{36}	0.0038	1.95×10^6	0.168
10^{-8}	4.72×10^{37}	0.0375	3.46×10^6	0.298
10^{-7}	4.72×10^{38}	0.375	6.15×10^6	0.530
2.66×10^{-7} (Edd.)	1.26×10^{39}	1.000	7.86×10^6	0.677

Table 2. Dependence on the accretion rate ($M = 10 M_{\odot}$).

Check: numerical integration of $4\pi r \sigma T^4(r)$ from R_{in} outward reproduces Eq. (3) to better than 0.01 % (ratio 1.000000); a numerical search for the maximum of Eq. (2) gives $r = 8.17 R_g$ and $T_{\max} = 0.488 T^*$.

4. Conclusions

(1) For a $10 M_{\odot}$ black hole the disk reaches $T_{\max} \approx 3.46$ million K, so the emission peaks in the soft X-ray band (≈ 0.8 keV). (2) The disk radiates only ≈ 3.8 % of the Eddington limit, which justifies the thin-disk approximation. (3) The disk is thin ($H/r \lesssim 0.02$), but the approximation fails near $\dot{M}_{\text{Edd}} \approx 2.7 \times 10^{-7} M_{\odot}/\text{yr}$. (4) The Keplerian frequency at the inner edge, ≈ 220 Hz, is of the order of the quasi-periodic oscillation frequencies observed in black-hole binaries. The model neglects Compton scattering and relativistic effects; the color temperature may reach ≈ 5.9 million K for $f \approx 1.7$ [5].

References

1. Shakura N.I., Sunyaev R.A. Black holes in binary systems. Observational appearance // Astron. Astrophys. 1973. Vol. 24. P. 337–355.
2. Novikov I.D., Thorne K.S. Astrophysics of black holes // Black Holes (Les Astres Occlus). New York: Gordon & Breach, 1973. P. 343–450.
3. Frank J., King A., Raine D. Accretion Power in Astrophysics. 3rd ed. Cambridge: Cambridge University Press, 2002.
4. Mitsuda K. et al. Energy spectra of low-mass binary X-ray sources observed from Tenma // Publ. Astron. Soc. Japan. 1984. Vol. 36. P. 741–759.
5. Shimura T., Takahara F. On the spectral hardening factor of the X-ray emission from accretion disks in black hole candidates // Astrophys. J. 1995. Vol. 445. P. 780–788.