

METHODOLOGICAL FOUNDATIONS FOR THE USE OF DIGITAL VISUALIZATION TOOLS IN TEACHING PROBABILITY THEORY IN HIGHER EDUCATION

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Abstract: This thesis examines the methodological foundations for the use of digital visualization tools in teaching probability theory in higher education. Particular attention is paid to the role of dynamic visualization, computer simulations, interactive graphs, and digital probability calculators in supporting students' understanding of random events, probability distributions, conditional probability, and statistical regularities. The study discusses how digital tools can transform abstract mathematical concepts into observable and experimentally testable models. Examples involving coin-toss simulations, binomial distributions, conditional probability, and normal distributions are presented to demonstrate the practical application of visualization-based instruction.

Key words: probability theory, higher education, digital visualization, interactive learning, mathematical simulation, GeoGebra, probability distribution, probabilistic reasoning, digital technologies.

Probability theory is an essential component of mathematics education in higher education because it provides a theoretical basis for statistics, data analysis, economics, engineering, computer science, artificial intelligence, and many other fields. Nevertheless, students often experience difficulties in understanding probability because many of its fundamental concepts are abstract and cannot be directly observed. Concepts such as random variables, independence, conditional probability, expected value, and probability distributions require students to combine mathematical reasoning with an understanding of uncertainty. Research in probability education has therefore emphasized the importance of different representations, simulations, technological tools, and visualization in developing students' probabilistic reasoning [1].

Traditional instruction frequently introduces probability through definitions, formulas, and manually solved problems. For example, students learn that the probability of obtaining a head when tossing a fair coin is

$$P(H) = 1/2.$$

Although the formula is simple, knowing it does not necessarily mean that students understand the relationship between theoretical probability and long-run experimental frequency. Digital simulation makes this relationship observable. Students can simulate 10, 100, 1,000, or 10,000 coin tosses and examine how the relative frequency of heads changes as the number of trials increases.

Modern research on teaching and learning probability identifies simulation and visualization as important areas because they can reduce the abstractness of probability and help learners investigate stochastic phenomena experimentally [2]. Therefore, the integration of digital visualization into university probability courses requires not only technological knowledge but also an appropriate methodological framework.

Digital visualization in probability education refers to the use of interactive graphs, simulations, diagrams, animations, and mathematical software to represent random processes and probability models visually. Its methodological importance lies in establishing connections among mathematical formulas,

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graphical representations, numerical results, and experimental observations. The methodological approach can be organized according to the following sequence:

Prediction → Simulation → Visualization → Mathematical explanation → Reflection.

At the prediction stage, students are encouraged to formulate an expectation before using technology. During simulation, they generate experimental data. Visualization enables them to identify patterns in the results. Mathematical explanation then connects these observations with probability theory, while reflection encourages students to compare their initial predictions with the obtained results. This structure is important because simply displaying an attractive graph does not guarantee mathematical understanding. Research on probability education emphasizes that technology provides significant opportunities for simulation, but students also need appropriate instructional support to interpret the underlying probability models [3].

Example 1. Coin-Toss Simulation

Consider the experiment of tossing a fair coin. The theoretical probability of obtaining heads is

$$P(H) = 0.5.$$

A teacher can first ask students:

“If we toss a coin 20 times, will exactly 10 tosses necessarily result in heads?”

Some students may incorrectly assume that theoretical probability guarantees an equal number of heads and tails in every small sample.

Using a digital simulation, students can conduct 20 virtual tosses and obtain, for example:

$$\text{Heads} = 13, \text{Tails} = 7.$$

The experimental probability is therefore

$$P(H) = 13/20 = 0.65.$$

Students can then increase the number of simulations to 1,000 or 10,000 trials. They can observe that the relative frequency tends to become more stable around 0.5 as the number of trials increases.

GeoGebra, for example, provides interactive probability resources that include coin-toss simulations and activities comparing theoretical and experimental probability. It also provides tools for exploring probability distributions [4].

The pedagogical value of this activity is not the calculation itself. Rather, students visually observe the distinction between theoretical probability and experimental relative frequency.

Another important methodological application concerns probability distributions. Students often find it difficult to interpret formulas for discrete random variables without graphical support. Suppose that a random variable X represents the number of heads obtained when a fair coin is tossed five times. Then

$$X \sim B(5, 0.5).$$

The probability of obtaining exactly three heads is

$$P(X = 3) = C(5,3)(0.5)^3(0.5)^2$$

and therefore

$$P(X = 3) = 10/32 = 0.3125.$$

A digital probability calculator can simultaneously display the values of the binomial distribution as bars. Students can change the parameters n and p and immediately observe changes in the distribution.

For example, they can compare:

$$B(5, 0.5), B(20, 0.5), \text{ and } B(100, 0.5).$$

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Instead of treating these distributions as three unrelated calculations, students can observe how increasing the number of trials affects the shape and concentration of the probability distribution. GeoGebra's Probability Calculator supports exploration of distributions including binomial and Poisson distributions, providing a practical environment in which mathematical calculations can be connected with visual representations [4].

Conditional probability represents another concept that can create difficulties for students. Consider the formula

$$P(A|B) = P(A \cap B) / P(B), \text{ provided } P(B) > 0.$$

Students may memorize this expression without developing an intuitive understanding of why the sample space changes after condition **B** is known.

For example, suppose that a group contains 60 students:

- 35 study English;
- 25 study mathematics;
- 15 study both subjects.

If a randomly selected student is already known to study mathematics, the probability that this student also studies English is

$$P(E|M) = 15/25 = 0.60.$$

A dynamic Venn diagram or probability tree can visually highlight the reduced sample space. Instead of considering all 60 students, learners see that the relevant reference group now consists only of the 25 mathematics students. Research comparing digital materials containing dynamic visualizations and feedback with comparable printed materials in lessons on conditional probability found benefits associated with the digital learning environment. The study involved 314 students and used a pretest-posttest control-group design [5]. Although such findings should not automatically be generalized to every higher-education context, they demonstrate the potential educational value of combining probability instruction with dynamic representations.

Digital visualization can also support university students in understanding continuous probability distributions. Consider a normally distributed variable:

$$X \sim N(100, 15^2).$$

Suppose students need to determine

$$P(X < 115).$$

After standardization,

$$Z = (115 - 100) / 15 = 1.$$

Therefore,

$$P(X < 115) \approx 0.8413.$$

A conventional solution may stop after obtaining this value. A digital visualization environment can additionally shade the area under the normal curve to the left of $x = 115$.

This visual representation helps students understand that probability for a continuous random variable corresponds to an **area under a probability density curve**, rather than simply being a number produced by a formula. Research examining instructional visualizations of the normal probability distribution has also highlighted the importance of visual scaffolds in supporting students' understanding of the concept. [5]

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The effective use of digital visualization in university probability courses should be based on several interconnected principles. First, the **principle of conceptual purposefulness** requires the teacher to select technology according to a specific mathematical objective. A simulation should be used because it makes a stochastic relationship observable, not merely because digital technology is available. Second, the **principle of multiple representation** requires students to move among formulas, numerical values, tables, simulations, and graphs. For instance, a binomial probability should first be expressed mathematically, then calculated, simulated, and finally interpreted graphically. Third, the **principle of active experimentation** requires students to modify parameters themselves. When exploring a binomial distribution, learners can change **n** and **p** and predict the resulting changes before viewing them.

Fourth, the **principle of interpretation** requires students to explain visual results mathematically. Questions such as “Why did the distribution become more concentrated?”, “Why is the experimental frequency approaching the theoretical probability?”, and “What does the shaded area represent?” transform visualization into mathematical reasoning.

Finally, the **principle of gradual transition from concrete to abstract representation** suggests that students can first explore a random phenomenon experimentally and visually before formalizing it through mathematical notation.

Consequently, digital visualization changes the methodological structure of a probability lesson. Instead of following only the traditional sequence

Definition → Formula → Example → Exercise,

a technology-supported lesson can follow:

Problem → Prediction → Digital experiment → Visualization → Mathematical model → Explanation → Generalization.

This approach creates stronger connections between theoretical probability and observable stochastic phenomena. It also encourages students to test hypotheses, analyze patterns, identify incorrect assumptions, and justify mathematical conclusions.

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