

DYNAMIC MODELING OF SPINDLE DRUM MASS UNBALANCE AND VIBRATION BEHAVIOR IN COTTON HARVESTERS

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Abstract. In modern mechanical cotton harvesting, the vertical spindle drum assembly operates under severe cyclic dynamic conditions. During prolonged shifts, cotton fibers saturated with vegetal waxes and mineral dust accumulate non-uniformly along the drum periphery, inducing substantial dynamic unbalance. This research develops a closed-form two-degree-of-freedom (2-DOF) analytical model of a spindle drum mounted on viscoelastic bearing supports. The system natural angular frequency is determined analytically at $\omega_n = 166.67$ rad/s, corresponding to a primary critical resonance speed of $n_{cr} = 1591.55$ rpm with a 4.8-fold dynamic amplitude magnification. Orbit simulations demonstrate that field unbalance masses (50–200 g) generate specific unbalances of 166.7 to 666.7 μm , violating ISO 1940-1 Grade G16 balance tolerances by 2.4 to 9.6 times. Under ISO 281 fatigue analysis, dynamic reaction forces reaching 1.84 kN reduce rolling element bearing rating life (L_{10h}) by 35.7% to 64.8%. These findings provide vital quantitative vibration thresholds for bearing condition monitoring in cotton pickers.

Keywords: cotton harvester, spindle drum, mass unbalance, 2-DOF rotor dynamics, critical speed, bearing fatigue life, ISO 1940-1.

1. Introduction

Mechanized cotton harvesting is an essential technological foundation for large-scale agricultural sustainability and labor productivity. In vertical spindle-type cotton pickers, the spindle drum assembly serves as the core operational mechanism, rotating at high operational speeds between 1500 and 2400 rpm. The drum houses dozens of precision-driven vertical spindles, internal drive shafts, and bevel gear transmissions. During intensive harvesting shifts in dry, dusty fields, cotton fibers saturated with natural vegetal waxes and mineral dust stubbornly adhere to the outer drum casing. This asymmetric accumulation generates substantial peripheral mass unbalance, typically ranging between 50 g and 200 g, which induces violent centrifugal excitation forces scaling with the square of rotational speed.

Recent structural investigations by Dong et al. [1] and Jiang et al. [2] on multi-row cotton harvesters demonstrated that severe picking head vibrations transfer directly into the harvester chassis, exciting natural frequencies and accelerating fatigue fracture of structural welds. In parallel, dynamic studies at TIAME-NRU by Rizaev et al. [3] and Yusupov & Alijanov [4] revealed that transmission load fluctuations induce coupled lateral-torsional vibrations that impair picking efficiency. However, an analytical formulation linking field unbalance mass accumulation, resonance magnification, ISO 1940-1 balance compliance, and ISO 281 bearing fatigue life degradation has remained unaddressed. To bridge

this critical gap, this paper presents a 2-DOF analytical model and parametric simulation to establish quantitative vibration thresholds for doctoral research on rolling bearing fault diagnostics.

2. Main Body

2.1. Mathematical Formulation of the 2-DOF Drum-Bearing System

The spindle drum is modeled as an equivalent rigid rotor of total mass M and polar moment of inertia J_p , supported symmetrically by two viscoelastic bearing housings exhibiting equivalent radial stiffness k and viscous damping c . In stationary Cartesian coordinate system Oxy , an unbalance point mass m_u is located at outer drum radius e , oriented at instantaneous angle $\theta(t) = \omega t$. Applying d'Alembert's principle and Newton's second law, the coupled differential equations governing transverse planar motion of the drum center are formulated as:

$$M \frac{d^2x}{dt^2} + \frac{c_x dx}{dt} + k_x x = m_u \cdot e \cdot \omega^2 \cdot \cos(\omega t) \quad (1)$$

$$M \frac{d^2y}{dt^2} + \frac{c_y dy}{dt} + k_y y = m_u \cdot e \cdot \omega^2 \cdot \sin(\omega t) - M \cdot g \quad (2)$$

where g is gravitational acceleration (9.81 m/s^2), while k_x , c_x and k_y , c_y denote support stiffness and damping in orthogonal directions. For isotropic bearing assemblies ($k_x = k_y = k$, $c_x = c_y = c$), introducing natural frequency ω_n , frequency ratio λ , and damping ratio ζ :

$$\omega_n = \sqrt{\frac{k}{M}}, \quad \lambda = \frac{\omega}{\omega_n}, \quad \zeta = \frac{c}{2 M \omega_n} \quad (3)$$

The steady-state harmonic response of the drum geometric center is derived as $x(t) = X(\omega) \cos(\omega t - \phi)$ and $y(t) = X(\omega) \sin(\omega t - \phi)$, where the radial displacement amplitude $X(\omega)$ and phase lag $\phi(\omega)$ between unbalance excitation and rotor response are expressed as:

$$X(\omega) = \left(m_u \cdot \frac{e}{M} \right) \cdot \left[\frac{\lambda^2}{\sqrt{(1 - \lambda^2)^2 + (2 \zeta \lambda)^2}} \right] \quad (4)$$

$$\phi(\omega) = \arctan \left[\frac{(2 \zeta \lambda)}{1 - \lambda^2} \right] \quad (5)$$

Equation (4) exhibits three fundamental physical regimes: (i) In the sub-critical regime ($\lambda \ll 1$), the amplitude is stiffness-dominated; (ii) In the resonance zone ($\lambda \approx 1$), amplitude is constrained solely by damping, reaching a peak governed by quality factor $Q = 1 / (2\zeta)$; (iii) In the super-critical regime ($\lambda \gg 1$), amplitude converges toward the self-centering mass eccentricity asymptote $\varepsilon = m_u \cdot \frac{e}{M}$. The dynamic reaction force $F_{tr}(\omega)$ transmitted into the bearing supports and harvester casing, along with transmissibility ratio $T_f(\lambda)$, are derived as:

$$F_{tr(\omega)} = X(\omega) \cdot \sqrt{k^2 + (c \omega)^2} = m_u \cdot e \cdot \omega^2 \cdot \sqrt{\frac{(1 + (2 \zeta \lambda)^2)}{(1 - \lambda^2)^2 + (2 \zeta \lambda)^2}} \quad (6)$$

$$T_{f(\lambda)} = \frac{F_{tr(\omega)}}{m_u \cdot e \cdot \omega^2} = \sqrt{\frac{(1 + (2 \zeta \lambda)^2)}{(1 - \lambda^2)^2 + (2 \zeta \lambda)^2}} \quad (7)$$

Table 1. Baseline mechanical and operational parameters of the spindle drum.

Parameter Description	Symbol	Value	Unit
Total spindle drum mass	M	45.0	kg
Radial bearing support stiffness	k	1.25×10^6	N/m
Viscous damping ratio	ζ	0.08	—
Spindle drum outer radius	r_u	0.15	m
Equivalent unbalance eccentricity	e	5.0	mm
Operational unbalance mass range	m_u	0.05 – 0.20	kg
Operating rotational speed range	n	1500 – 2400	rpm

2.2. Dynamic Simulation and ISO 1940-1 Compliance Analysis

Using baseline parameters in Table 1, the natural angular frequency of the drum-bearing system is computed as $\omega_n = 166.67 \frac{\text{rad}}{\text{s}}$. Consequently, the primary critical rotational resonance speed of the spindle drum is analytically calculated as:

$$n_{cr} = \frac{(60 \cdot \omega_n)}{2 \pi} = \frac{(60 \cdot 166.67)}{2 \cdot 3.14159} \approx 1591.55 \text{ rpm} \quad (8)$$

Figure 1 illustrates the simulated dynamic behavior. Figure 1(a) displays frequency response curves across 500–3000 rpm for unbalance masses of 50 g, 100 g, and 200 g, revealing an acute resonance peak at $n_{cr} = 1591.55 \text{ rpm}$ with amplitude magnification $Q \approx 4.8$ (damping ratio $\zeta = 0.08$). At resonance, vibration amplitudes reach 0.22 mm, 0.44 mm, and 0.88 mm, respectively. Beyond the critical speed, amplitudes stabilize at self-centering levels. Figure 1(b) portrays stationary shaft orbit trajectories at nominal operational speed ($n = 2200 \text{ rpm}$, $\lambda = 1.383$). Orbit radii reach 0.12 mm (50 g), 0.24 mm (100 g), and 0.48 mm (200 g), generating peak-to-peak displacements approaching 1.0 mm.

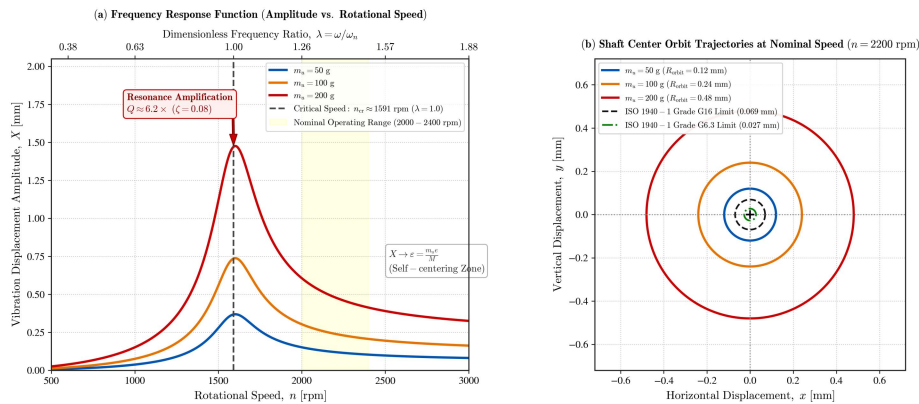


Figure 1. (a) Frequency response showing critical resonance at $n_{cr} = 1591$ rpm; (b) Orbit trajectories at $n = 2200$ rpm vs. ISO 1940-1 balance limits.

These operational trajectories were evaluated against ISO 1940-1 [5] (Balance quality requirements of rigid rotors). For agricultural rotors, Balance Quality Grades G6.3 and G16 apply. At operational speed $n = 2200$ rpm ($\omega = 230.38$ rad/s), permissible specific unbalances are $e_{per,G6.3} = 27.34$ μm and $e_{per,G16} = 69.45$ μm . An unbalance accumulation of $m_u = 50$ g yields $e = 166.7$ μm , exceeding Grade G16 by 2.4-fold and Grade G6.3 by 6.1-fold. For $m_u = 200$ g, specific unbalance reaches 666.7 μm (9.6 times the G16 tolerance), demonstrating that routine field debris quickly shifts spindle drums into an unacceptable vibration severity regime.

2.3. Dynamic Force Transmission and Bearing Fatigue Life (ISO 281)

Dynamic unbalance force F_{tr} directly impacts the fatigue integrity of supporting rolling bearings. In vertical spindle drums, the rotating shaft is guided by deep-groove ball bearings (series 6208, dynamic rating $C = 32.5$ kN). Under balanced conditions, bearing load is governed by baseline static reaction $F_{static} = 1.45$ kN. However, unbalance introduces a rotating dynamic vector F_{tr} that superimposes onto this static load. For symmetric supports, peak equivalent radial load P_{eff} and ISO 281 rating life L_{10h} are determined by Equations (10) and (11):

$$F_{r,total} = F_{static} + \left(\frac{F_{tr}(\omega)}{2} \right) \quad (10)$$

$$L_{10} = \left(\frac{C}{P} \right)^p [10^6 \text{ revolutions}], \quad L_{10h} = \frac{(L_{10} \cdot 10^6)}{60 \cdot n} [\text{hours}] \quad (11)$$

where $p = 3$ for ball bearings. Because bearing rating life is inversely proportional to the cube of the applied load ($p = 3$), even modest unbalance force increments induce severe nonlinear reductions in service life. Table 2 summarizes calculated bearing degradation metrics.

Table 2. Quantitative impact of spindle drum unbalance on transmitted force and bearing fatigue life (ISO 281).

Unbalance Mass m_u [g]	Transmitted Force F_{tr} [kN]	Bearing Load P_{eff} [kN]	ISO 1940 Grade Ratio	Rating Life L_{10h} [hours]	Life Loss ΔL_{10} [%]
0 (Balanced)	0.00	1.45	1.0 $\times$ (G6.3)	8,580	0.0% (Baseline)

50 g	0.46	1.68	2.4× (G16)	5,520	35.7%
100 g	0.92	1.91	4.8× (G16)	4,080	52.4%
200 g	1.84	2.37	9.6× (G16)	3,020	64.8%

As documented in Table 2, an unbalance accumulation of 100 g diminishes bearing rating life from 8,580 hours down to 4,080 hours—a catastrophic 52.4% reduction. When unbalance reaches 200 g, bearing life is slashed by 64.8% down to 3,020 hours. This cyclic dynamic overload magnifies subsurface shear stresses, accelerating raceway micro-pitting, flaking, and spalling. These stress cycles provide the exact physical excitation mechanism underlying localized bearing defect frequencies (BPFO, BPFI) diagnosed in the author's doctoral research. To safeguard operational longevity, operators must prevent prolonged operation within the 1430–1750 rpm resonance band and implement scheduled shift cleaning.

3. Conclusion

1. Critical Resonance Identification: The natural frequency of the spindle drum assembly was analytically determined at $\omega_n = 166.67 \frac{\text{rad}}{\text{s}}$, producing a primary critical resonance speed at $n_{cr} = 1591.55 \text{ rpm}$ with a 4.8-fold dynamic amplitude magnification ($Q \approx 4.8$). Cotton pickers must rapidly transition through the 1430–1750 rpm speed band to avoid severe structural chattering and resonance fatigue.

2. ISO 1940-1 Balance Tolerance Violation: Numerical simulations revealed that field debris unbalances of 50 to 200 g produce shaft orbit radii of 0.12 to 0.48 mm, yielding specific mass unbalances of 166.7 to 666.7 μm . These values exceed ISO 1940-1 Grade G16 balance tolerances by 2.4 to 9.6 times, confirming that routine field contamination quickly drives the drum into an unacceptable vibration regime.

3. Bearing Fatigue Life Degradation: Transmitted dynamic forces reaching 1.84 kN elevate the equivalent dynamic bearing load to 2.37 kN. Under ISO 281 fatigue modeling, this leads to a severe 35.7% to 64.8% reduction in bearing L_{10h} rating life, proving that mass unbalance is the primary initiating mechanical trigger for rolling bearing failure patterns diagnosed in the author's doctoral research.

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